

## Equivalent fractions

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How many fractions are there equivalent to :

$$1) \quad \frac{1}{5} = \frac{2}{10} = \frac{5}{25} = \frac{3}{15} = \frac{4}{20}$$

$$2) \quad \frac{2}{5} = \frac{4}{10} = \frac{8}{20} = \frac{10}{25} = \frac{20}{50} = \frac{40}{100}$$

## Cancelling Fractions

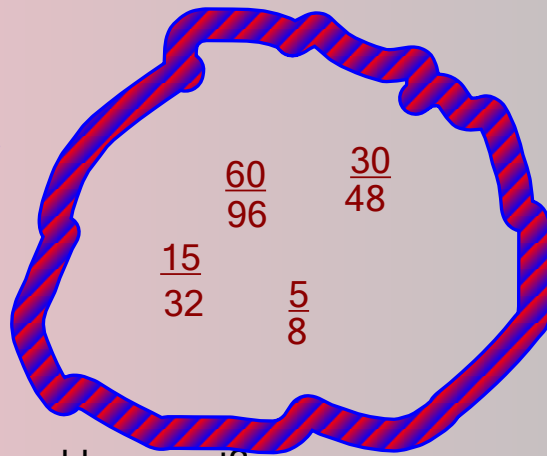
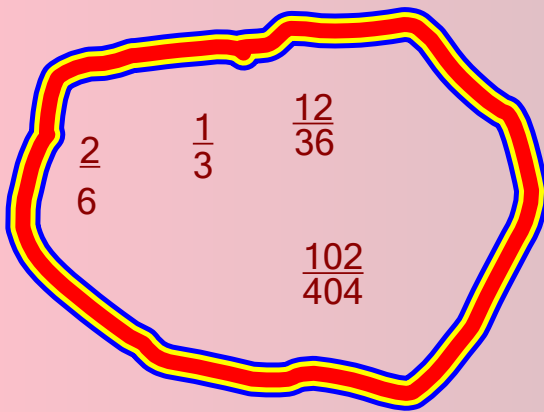
$$\frac{\cancel{4}^2}{\cancel{12}^6} = \frac{1}{3}$$

$$\frac{\cancel{24}^{12}}{\cancel{100}^{50}} = \frac{6}{25}$$

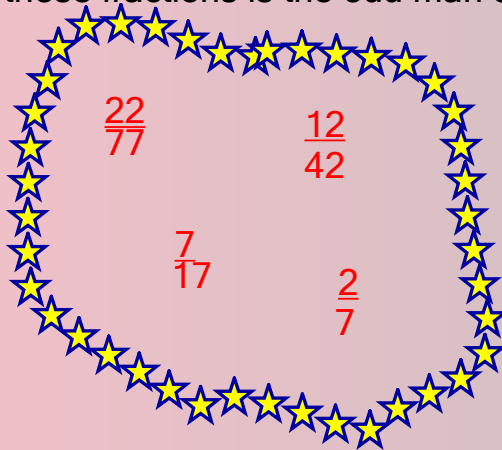
Cancel these fractions to their lowest terms

$$\frac{20}{45} = \frac{4}{9} \quad \frac{5}{45} = \frac{1}{9} \quad \frac{40}{90} = \frac{4}{9}$$

$$\frac{27}{42} = \frac{9}{14} \quad \frac{16}{96} = \frac{1}{6} \quad \frac{120}{600} = \frac{1}{5}$$



Which of these fractions is the odd man out?



Ordering fractions

There are two main ways to order fractions:

- if you can use a calculator, change them to decimals.
- use equivalent fractions to compare.

Put these fractions in order of size, starting with the smallest:

1)  $\frac{5}{6}, \frac{2}{3}, \frac{3}{10}, \frac{1}{2}$   $\frac{3}{10}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$  no calculator  
 $\frac{5}{30}, \frac{20}{30}, \frac{9}{30}, \frac{15}{30}$

2)  $\frac{4}{33}, \frac{8}{66}, \frac{6}{11}, \frac{36}{66}, \frac{1}{2}, \frac{33}{66}, \frac{5}{6}, \frac{55}{66}$  no calculator  
 $\frac{1}{4}, \frac{2}{5}, \frac{1}{6}, \frac{2}{7}$   $2 \div 7 = 0.285...$   $\frac{4}{33}, \frac{1}{2}, \frac{6}{11}, \frac{5}{6}$

3)  $\frac{1}{4}, \frac{2}{5}, \frac{1}{6}, \frac{2}{7}$   $0.4$   $\frac{1}{6}, \frac{1}{4}, \frac{2}{7}, \frac{2}{5}$

4)  $\frac{5}{8}, \frac{3}{7}, \frac{4}{9}, \frac{3}{10}$   $\frac{3}{10}, \frac{3}{7}, \frac{4}{9}, \frac{5}{8}$

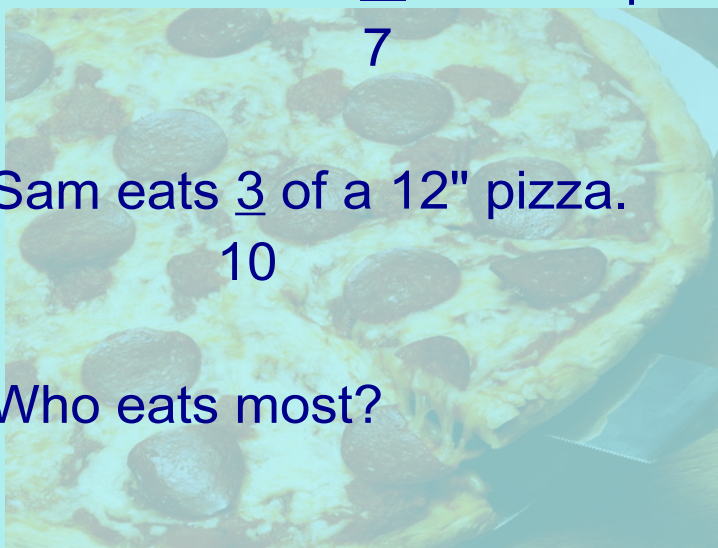
5)  $\frac{7}{11}, \frac{4}{15}, \frac{6}{12}, \frac{5}{7}$   $0.3, 0.62, 0.4$   $\frac{4}{15}, \frac{6}{12}, \frac{7}{11}, \frac{5}{7}$

6)  $\frac{1}{6}, \frac{2}{9}, \frac{5}{12}, \frac{7}{15}$   $0.16, 0.2$   $\frac{1}{6}, \frac{2}{9}, \frac{5}{12}, \frac{7}{15}$  Dolphin game

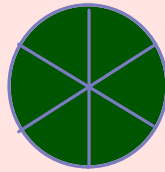
Lawrence eats  $\frac{2}{7}$  of a 12" pizza.

Sam eats  $\frac{3}{10}$  of a 12" pizza.

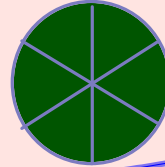
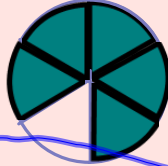
Who eats most?



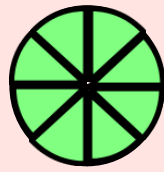
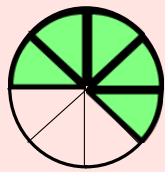
Improper Fractions and mixed numbers



$$\frac{7}{6} = 1\frac{1}{6}$$



$$\frac{11}{6} = 1\frac{5}{6}$$




$$\frac{13}{8} = 1\frac{5}{8}$$

Draw a diagram to show  $1\frac{2}{3} = 1\frac{5}{3}$

Draw a diagram to show  $\frac{7}{5} = 1\frac{2}{5}$

American currency is in cents:

dimes (10c), 

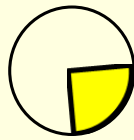
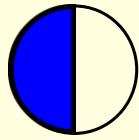
quarters (25c) 

and dollars (100c).

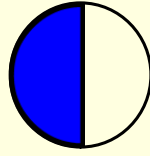
If I have 11 quarters, how much money is that?  $\$2.75$



Adding Fractions with different denominators



$$\frac{1}{2} + \frac{1}{4}$$
$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$



$$\frac{1}{2} + \frac{2}{6} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$\frac{1}{4} + \frac{1}{8}$$

$$\frac{1}{3} + \frac{1}{6}$$

1.  $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

2.  $\frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

3.  $\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}$

4.  $\frac{2}{5} + \frac{3}{15} = \frac{6}{15} + \frac{3}{15} = \frac{9}{15}$

5.  $\frac{3}{4} + \frac{1}{12} =$

6.  $\frac{1}{2} + \frac{1}{3} =$

$\frac{2}{5} + \frac{3}{7} =$

How to add and subtract fractions without going to pieces

1. Find a common denominator, the lower the better.
2. Using your common denominator make equivalent fractions.
3. Add (or subtract) the numerators.
4. Simplify.

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

$$\frac{2}{5} + \frac{1}{8} = \frac{\quad}{40} + \frac{\quad}{40}$$

$$\frac{1}{3} + \frac{1}{5}$$

$$\frac{3}{5} + \frac{5}{9}$$

$$\frac{2}{3} + \frac{1}{5}$$

$$\frac{5}{6} + \frac{1}{9}$$

Danielle eats  $\frac{3}{8}$  of a bar of

chocolate. Tamara eats  $\frac{2}{5}$  of the bar?

How much is left for Georgia?

$$\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40} \quad \frac{9}{40}$$

$$\frac{1}{3} + \frac{2}{6} + \frac{4}{12}$$

$$\frac{6}{18} + \frac{12}{36} + \frac{24}{72}$$

Find 3 fractions that have a sum of 1,  
but all have different denominators.

$$\frac{1}{2} + \frac{2}{8} + \frac{1}{4}$$

$$\frac{1}{4} + \frac{1}{8} + \frac{5}{8}$$

$$\frac{1}{4} + \frac{1}{8} + \frac{20}{32}$$

## Finding a fraction of an amount.

$\frac{5}{8}$  of 88 means find one eighth, then find 5 of those

$$\frac{1}{8} \text{ of } 88 = 11$$

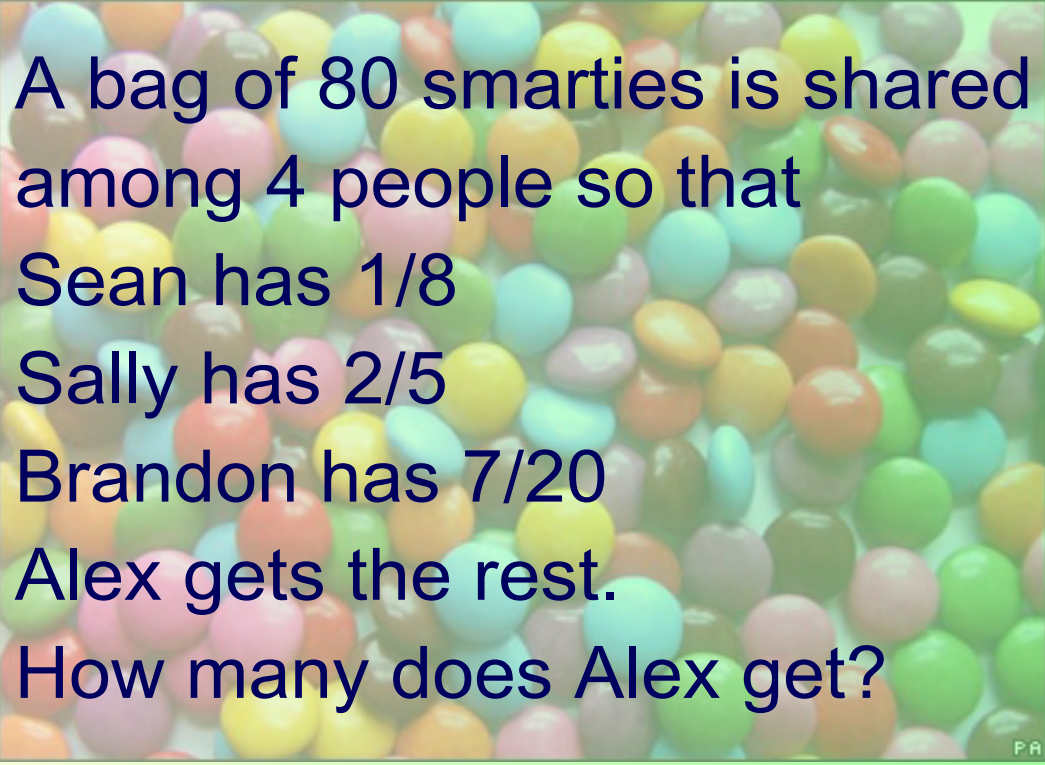
$$\frac{5}{8} \text{ of } 88 = 55$$

work out:

$$\frac{3}{7} \text{ of } 35 = 15$$

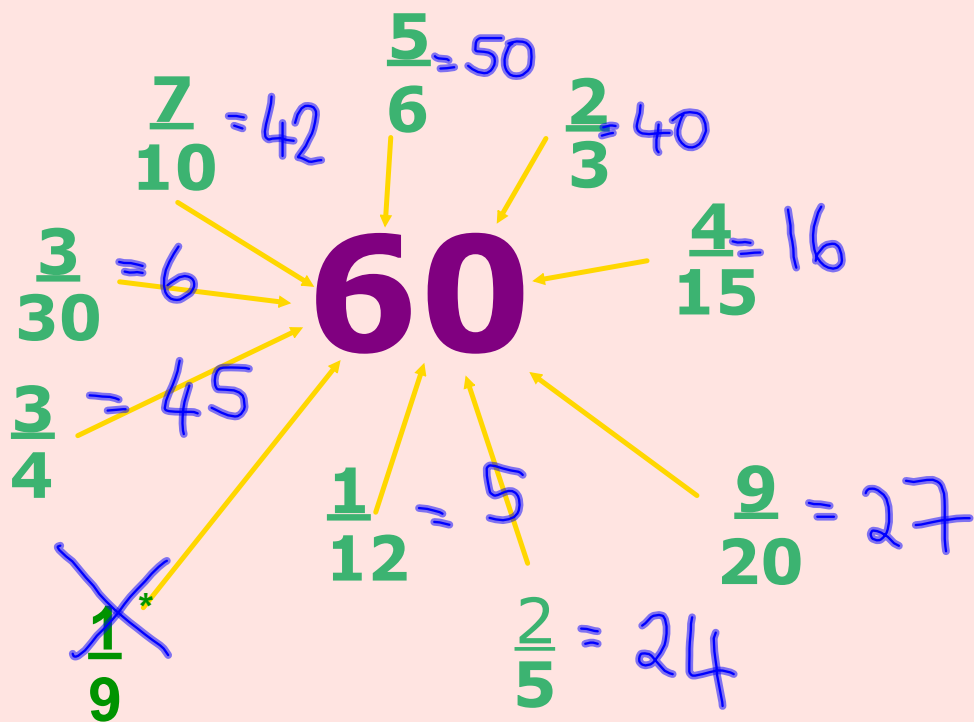
$$\frac{4}{5} \text{ of } 45 = 36$$

$$\frac{2}{9} \text{ of } 36 = 8$$



A bag of 80 smarties is shared among 4 people so that  
Sean has  $\frac{1}{8}$   
Sally has  $\frac{2}{5}$   
Brandon has  $\frac{7}{20}$   
Alex gets the rest.  
How many does Alex get?

Work out:



page 145 A2, B2-B8 evens only, C1-C4

## Cancel when you can

$$\frac{\overset{6}{\cancel{12}}}{\cancel{20}_{10}} = \frac{\overset{6}{\cancel{10}}^3}{5} = \frac{3}{5}$$

$$\frac{12}{20} = \frac{\cancel{2 \times 2 \times 3}}{\cancel{2 \times 2 \times 5}} = \frac{3}{5}$$

Write the following as product of their primes and cancel:

$$\frac{8}{12} = \frac{2}{3}$$

$$\frac{24}{42} = \frac{4}{7}$$

### Multiplying Fractions

This is the easiest of all the fraction questions.  
Multiply the numerators together.  
Multiply the denominators together.  
That's it!

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

$$\frac{21}{12} = \frac{7}{4}$$

If you have a whole number, change it to an improper fraction:

$$2\frac{1}{3} \times \frac{3}{4} = \frac{7}{3} \times \frac{3}{4} = \frac{7}{4} = 1\frac{3}{4}$$

A shortcut:

$$\frac{2}{3} \times \frac{6}{10} = \frac{2}{5}$$

$$\frac{3}{20} \times \frac{5}{9} = \frac{1}{12}$$

$$\frac{13}{20} \times \frac{5}{39}$$

Cancel where you can

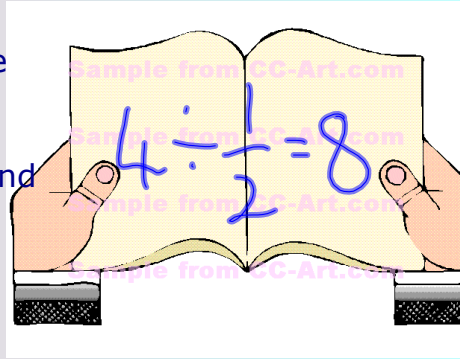
$$\frac{\cancel{45}^9}{\cancel{70}_2} = \frac{9}{14}$$
$$\frac{45}{120} = \frac{9}{24}$$
$$\frac{42}{70} = \frac{6}{10} = \frac{3}{5}$$
$$\frac{\cancel{39}^1}{\cancel{390}_6} = \frac{1}{10}$$
$$\frac{\cancel{450}^5}{\cancel{720}_8} = \frac{5}{8}$$

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page 150 E1-E6, F1 - F8

## Dividing a whole number by a fraction

Rebecca and Sally have forgotten their books again!  
I have 4 spare pages and I divide each in half.  
How many pages do I have?



$$3 \div \frac{1}{5} = 15$$



I have 3 chocolate cakes and split each into 5ths.  
How many pieces do I have?

To divide by a fraction turn the fraction upside down and multiply

$$5 \div \frac{1}{4} = 5 \times \frac{4}{1} = 20$$

$$5 \div \frac{1}{3} = 5 \times \frac{3}{1} = 15$$

$$6 \div \frac{1}{3} = 6 \times \frac{3}{1} = 18$$

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = 9$$

$$8 \div \frac{2}{5} = \cancel{8}^4 \times \frac{5}{\cancel{2}} = 20$$

page 284 C3-C9 **odds only**

## Dividing by a fraction

This is easy if you can remember the rule:  
Turn the second fraction upside down and  
multiply!

$$\frac{1}{4} \div \frac{1}{4} = 1 \quad \frac{1}{4} \times \frac{4}{1} = 1$$

How many quarters are there in a half?

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

$$\begin{array}{l} \frac{1}{2} \div \frac{1}{8} \\ = \frac{1}{2} \times \frac{8}{1} \\ = \frac{8}{2} = 4 \end{array} \quad \begin{array}{l} \frac{1}{5} \div \frac{1}{10} \\ = \frac{1}{5} \times \frac{10}{1} \\ = \frac{10}{5} = 2 \end{array} \quad \begin{array}{l} \frac{3}{4} \div \frac{1}{4} \\ = \frac{3}{4} \times \frac{4}{1} = 3 \end{array}$$

$$\begin{array}{l} \frac{5}{6} \div \frac{1}{12} \\ = \frac{5}{6} \times \frac{12}{1} \\ = 10 \end{array} \quad \begin{array}{l} \frac{3}{7} \div \frac{6}{28} \\ = \frac{3}{7} \times \frac{28}{6} \\ = 2 \end{array}$$

Donald has a big box of sweets.  
He gives  $\frac{2}{5}$  to Nathaniel.  
Nathaniel divides his into 4  
groups. What fraction is each  
group of the original?



### Converting fractions to decimals

Which fractions do you know the decimal equivalent of?

Terminating decimals	Recurring Decimals
$0.75 = \frac{3}{4}$	$\frac{1}{3} = 0.\dot{3}$
$0.1 = \frac{1}{10}$	$\frac{2}{18} = 0.\dot{1}$
$0.25 = \frac{1}{4}$	$\frac{8}{9} = 0.\dot{8}$
$0.5 = \frac{1}{2}$	$\frac{1}{9} = 0.\dot{1}$
$0.3 = \frac{3}{10}$	$\frac{2}{9} = 0.\dot{2}$
$0.125 = \frac{1}{8}$	$\frac{2}{3} = 0.\dot{6}$
	$\frac{1}{18} = 0.\dot{0}5$



Investigate fractions with a denominator of 7

Investigate fractions with a denominator of 9.

Try to find a rule for fractions which terminate.  
Test it out on the fractions:

### Converting fractions to decimals by hand

$$\frac{1}{12} = 1 \div 12$$

$$\frac{5}{6} = 5 \div 6 = 0.8\dot{3}$$

$$\frac{4}{9} = 0.\dot{4}$$

$$\begin{array}{r} 0.0\dot{8}3 \\ 12 \overline{) 1.0000} \\ \underline{12} \phantom{00} \\ 10 \phantom{00} \\ \underline{10} \phantom{00} \\ 44 \\ \underline{44} \\ 0 \end{array}$$

$$\begin{array}{r} 0.8\dot{3} \\ 6 \overline{) 5.00000} \\ \underline{52} \phantom{000} \\ 22 \\ \underline{22} \\ 0 \end{array}$$

$$\frac{6}{11} = 0.\dot{5}4$$

## Bus Stop Division

- $496 \div 8$   
$$\begin{array}{r} 62 \\ 8 \overline{) 496} \\ \underline{48} \phantom{0} \\ 16 \\ \underline{16} \\ 0 \end{array}$$
- $174 \div 3$   
$$\begin{array}{r} 58 \\ 3 \overline{) 174} \\ \underline{15} \phantom{0} \\ 24 \\ \underline{24} \\ 0 \end{array}$$
- $53724 \div 6$   
$$\begin{array}{r} 8954 \\ 6 \overline{) 53724} \\ \underline{48} \phantom{00} \\ 57 \phantom{0} \\ \underline{54} \phantom{0} \\ 32 \phantom{0} \\ \underline{30} \phantom{0} \\ 24 \\ \underline{24} \\ 0 \end{array}$$
- $2850 \div 6$   
$$\begin{array}{r} 475 \\ 6 \overline{) 2850} \\ \underline{24} \phantom{00} \\ 45 \phantom{0} \\ \underline{42} \phantom{0} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Can this recurring decimal be expressed as a fraction?

0.34343434

$$\begin{array}{r}
 \text{let } x = 0.343434\dots\dots \\
 100x = 34.343434\dots\dots \\
 x = 0.343434\dots\dots \\
 \hline
 99x = 34.000000 \\
 x = \frac{34}{99}
 \end{array}$$

0.454545454545....

$$\begin{array}{r}
 \text{let } x = 0.454545\dots\dots \\
 100x = 45.4545 \quad \} \text{---} \\
 x = 0.4545 \quad \} \\
 99x = 45 \\
 x = \frac{45}{99} = \frac{5}{11}
 \end{array}$$

0.678678678678...

$$\begin{array}{r}
 \text{let } x = 0.678678 \\
 1000x = 678.678 \\
 x = 0.678 \\
 \hline
 999x = 678 \\
 x = \frac{678}{999} = \frac{226}{333}
 \end{array}$$

0.405140514051....

$$\begin{array}{r}
 \text{let } x = 0.4051405 \\
 10000x = 4051.4051 \quad \} \text{---} \\
 9999x = 4051 \quad \} \\
 x = \frac{4051}{9999}
 \end{array}$$

### Recurring Decimals to Fractions

Express 0.5555... as a fraction

$$\begin{array}{r} \text{Let } x = 0.555 \\ 10x = 5.555 \\ \underline{x = 0.555} \\ 9x = 5 \end{array} \quad \left. \vphantom{\begin{array}{r} 10x \\ x \end{array}} \right\} -$$
$$9x = 5 \quad x = \frac{5}{9}$$

Examples:

Express as a fraction:

- 1) 0.373737...  $x = \frac{37}{99}$
- 2) 0.676767  $x = \frac{67}{99}$
- 3) 0.76876876876  $x = \frac{768}{999}$
- 4) 0.4444444  $x = \frac{4}{9}$

Show that  $0.676767 = \frac{67}{99}$

Show that  $0.546546546 = \frac{182}{333}$

$$\begin{array}{r} \text{Let } x = 0.546546\dot{5} \\ 1000x = 546.546\dot{5} \\ \underline{x = 0.546\dot{5}} \\ 999x = 546 \end{array} \quad \left. \vphantom{\begin{array}{r} 1000x \\ x \end{array}} \right\} - \begin{matrix} (11) \\ (11) \end{matrix}$$
$$x = \frac{546}{999} = \frac{182}{333}$$

## Fractions and Percentages

$\frac{5}{6}$  of a school day is spent in lessons.

What percentage of the day is spent in lessons?

$$5 \div 6 \times 100 = 83.3\%$$

To change a fraction to percentage divide the numerator by denominator and  $\times 100$

72% of students like sport.

$$\frac{72}{100} = \frac{36}{50} = \frac{18}{25}$$

What fraction of the students like sport?

### Converting between fractions and percentages

Fraction to percent:

Percent to fraction:

## Solving problems with fractions.

1. A cake is divided up between 3 people.

Chris has two fifths, Danny has two sevenths and Nathaniel has the rest.

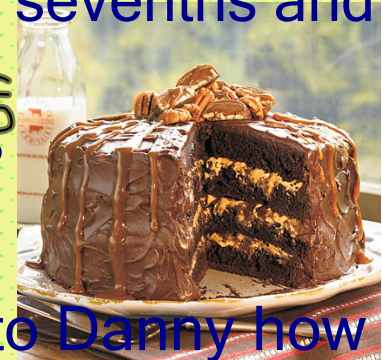
a) How much does Nathaniel get?

b) Who has the biggest piece?

c) If Nathaniel gave half his piece to Danny how much would they each get?

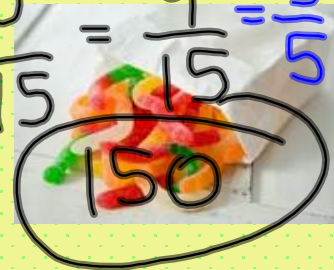
$$\frac{2}{5} + \frac{2}{7} = \frac{14}{35} + \frac{10}{35} = \frac{24}{35}$$

$$\frac{11}{70} + \frac{20}{70} = \frac{31}{70}$$



2. A bag of sweets is shared between 4 people. Danielle has 20%, Georgia has one fifteenth, Tamara has one third and Sarah has 60 sweets. How many sweets did they have?

$$\frac{1}{5} + \frac{1}{15} + \frac{1}{3} = \frac{3}{15} + \frac{1}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$$
$$\frac{2}{5} = \frac{6}{15} \text{ with } 60$$



3. A school day of 6 hours is divided into modules of three-quarters of an hour. How many modules are there in a day?

understand equivalent fractions, simplifying a fraction by cancelling all common factors  
order fractions by rewriting them with a common denominator  
calculate a given fraction of a given quantity, expressing the answer as a fraction  
express a given number as a fraction of another  
add and subtract fractions by writing them with a common denominator  
perform short division to convert a simple fraction to a decimal  
addition, subtraction, multiplication and division of mixed numbers  
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction  
distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals)  
convert a recurring decimal to a fraction  
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction  
convert simple fractions of a whole to percentages of the whole and vice versa  
use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer