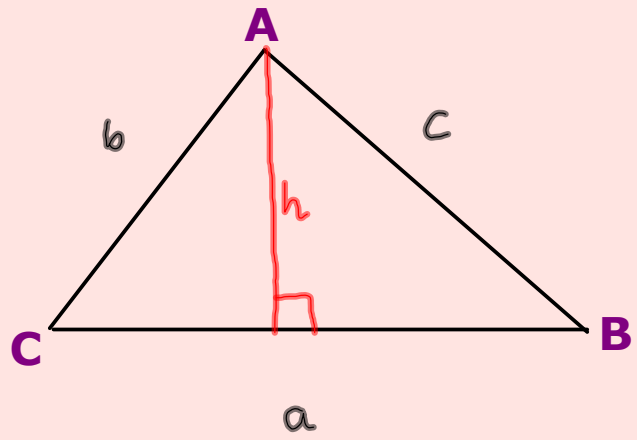
Area of a triangle without the height.



Prove that the area of this triangle is 1/2 ab sinC.
Find another way of expressing the area.

Hint: draw in the height.

Area of $\Delta = \frac{1}{2}$ base $\times$ ht

$= \frac{1}{2} a h$

We need to show $h = b \sin C$

Using the diagram $\sin C = \frac{h}{b}$

$b \sin C = h$

$\therefore$ Area of $\Delta = \frac{1}{2} a b \sin C$

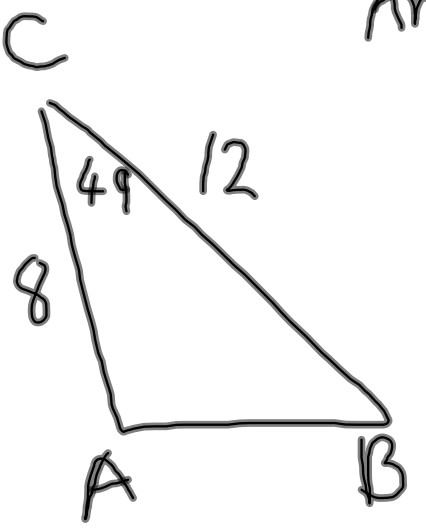
In order to use this formula, what info do you need?

You need an angle and its two adjacent sides

page 472 E4and E5

p 472

E4a)

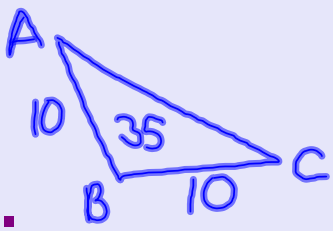


$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \cdot 12 \cdot 8 \cdot \sin 49$

$= 36.2\ cm^2$

Suppose triangle ABC has area = $35cm^2$
where AB = 10cm
and BC =10cm
and angle B is obtuse.



Find the size of angle B.

$\text{Area} = \frac{1}{2}ac\sin B$

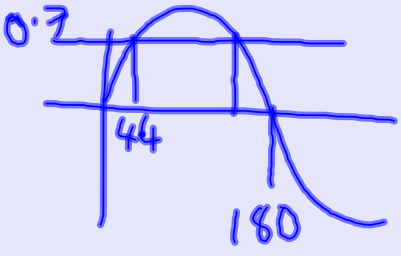
$35 = \frac{1}{2} \cdot 10 \cdot 10 \sin B$

$70 = 100 \sin B$

$0.7 = \sin B$

$B = 44.4^\circ$

But $\hat{B}$ is obtuse so $B = 135.6^\circ$

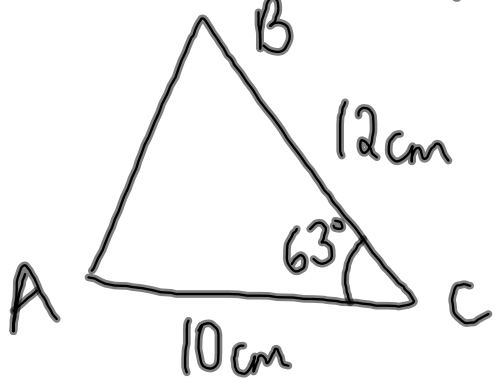


page 472

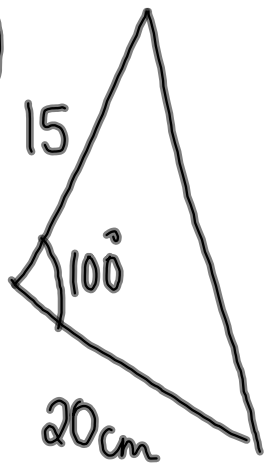
E 5 + F 6

Find the area of the Δ

i)

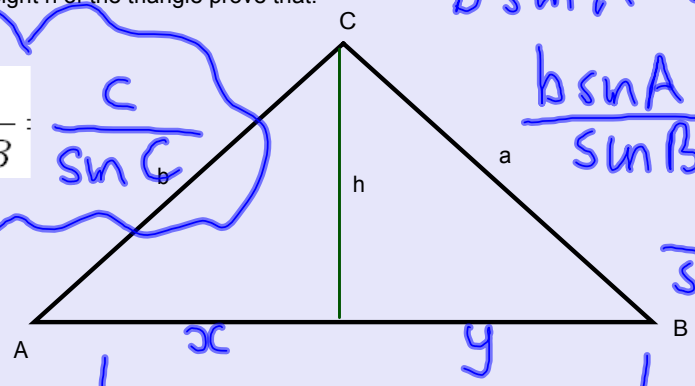


ii)



Consider a triangle ABC.
By constructing the height h of the triangle prove that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$b \sin A = a \sin B$

$\frac{b \sin A}{\sin B} = a$

$\frac{b}{\sin B} \sin A = a$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\sin A = \frac{h}{b}$

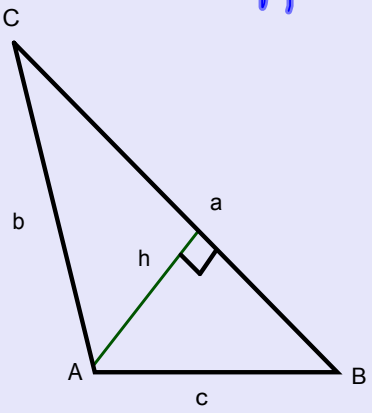
$b \sin A = h$

$\sin B = \frac{h}{a}$

$a \sin B = h$

Using the triangle ABC prove that:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$



Draw a triangle ABC and prove that:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

The Sine Rule

The Sine (and Cosine) Rule can be used in non right-angled triangles.

The Sine Rule states that:

For a side

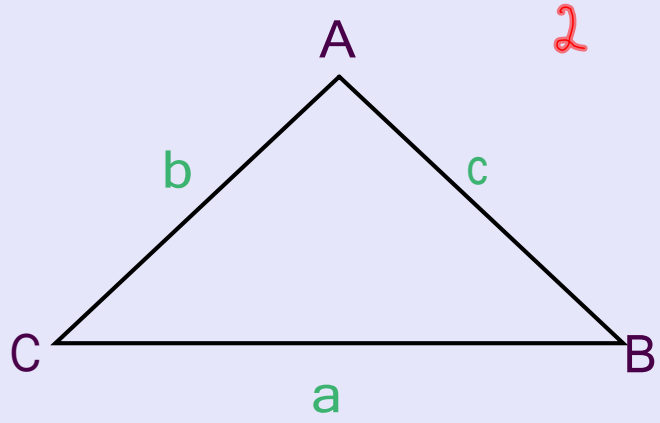
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

For an angle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

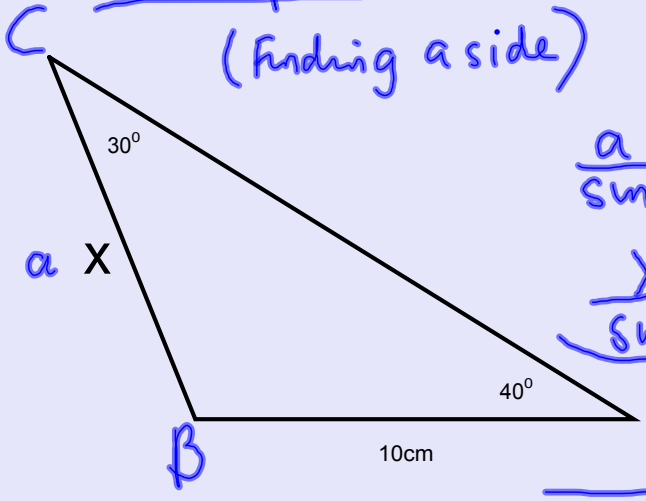
where A,B,C are the three vertices of the triangle, and a,b,c the lengths of their corresponding sides.

$\frac{1}{2} = \frac{2}{4}$ $\frac{2}{1} = \frac{4}{2}$



Example

(Finding a side)



$\frac{a}{\sin A} = \frac{c}{\sin C}$

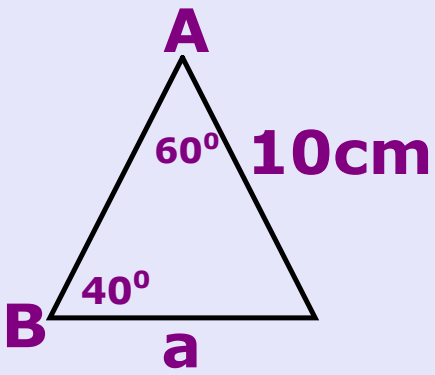
$\frac{X}{\sin 40} = \frac{10}{\sin 30}$

$X = \frac{10}{\sin 30} \times \sin 40$

$X = 12.9\text{cm}$

The Sine Rule

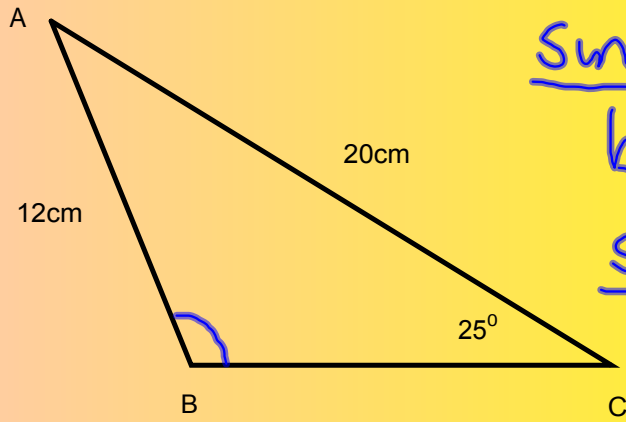
The Sine Rule is useful for finding a side or angle. You will need one pair of related data (angle and side) and one other angle or side.



You can use the info to find a.

Finding an angle using the sine rule

Example Find angle ABC



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

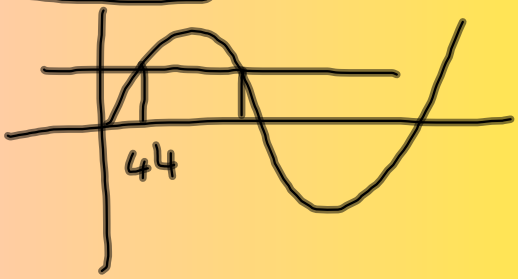
$$\frac{\sin B}{20} = \frac{\sin 25}{12}$$

$$\sin B = \frac{20 \sin 25}{12}$$

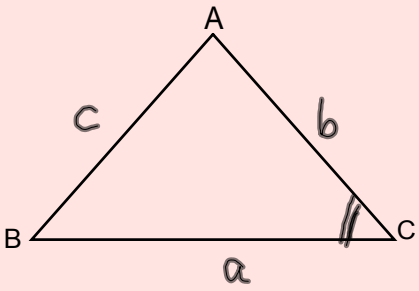
$$B = 44.8^\circ$$

$$\text{obtuse } \hat{B} = 135.2^\circ$$

page 468: C1-C3



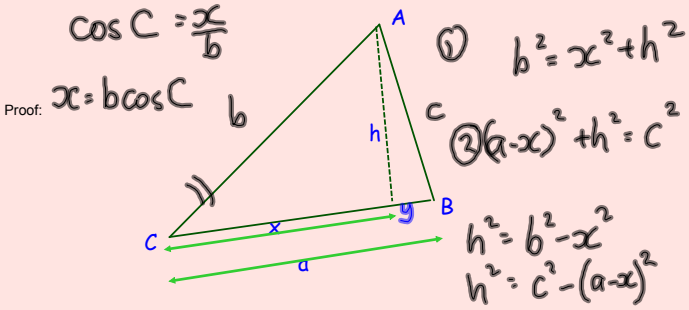
The Cosine Rule



The Cosine Rule is useful when you have been given two sides and the included angle.
The Cosine Rule states that:

$c^2 = a^2 + b^2 - 2ab\cos C$

Proof:



Use Pythagoras' theorem to find two equivalent expressions for h.

$b^2 - x^2 = c^2 - (a-x)^2$
$b^2 - x^2 = c^2 - a^2 + 2ax - x^2$ $\quad (a-x)(a-x) = a^2 - 2ax + x^2$
$b^2 = c^2 - a^2 + 2ax$
$b^2 = c^2 - a^2 + 2ab\cos C$
$b^2 + a^2 - 2ab\cos C = c^2$

QED

This is the formula to use if you want to find the side c.

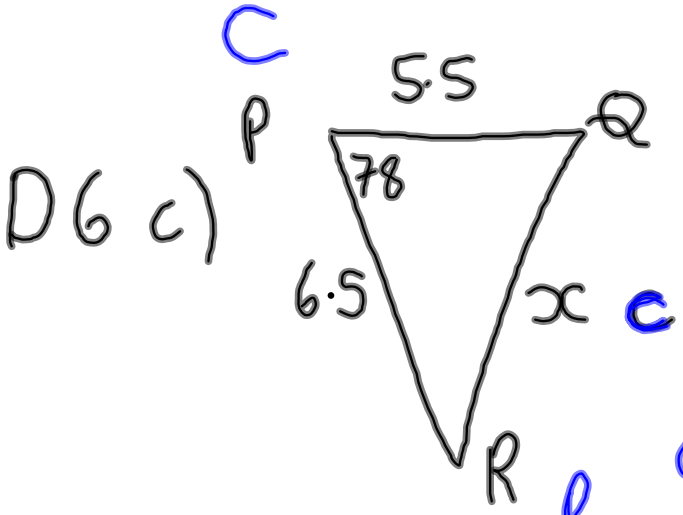
Rearrange the equation so that you have cos C as the subject.

$c^2 = a^2 + b^2 - 2ab\cos C$
$2ab\cos C = a^2 + b^2 - c^2$
$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

page 470: D6 onwards

Homework: Mixed questions (F1-F7) pages 473,474.
To be handed in on Friday. Tuesday

D6 c)



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 5.5^2 + 6.5^2 - 2 \times 5.5 \times 6.5 \cos 78$

$c^2 = 57.63$

$c = 7.59\text{cm}$

Attachments

 sin and y=0.3.agg