

# Quadratics

## Multiplying out.....

$$(2x - 4)(3x + 1) = 6x^2 + 2x - 12x - 4 \\ = 6x^2 - 10x - 4$$

$$(4x - 1)(4x + 1) = 16x^2 - 1 = ( \quad )( \quad )$$

$\underbrace{\quad\quad\quad}$   
 $\uparrow \quad \uparrow$

$$(3x + 8)^2 = (3x + 8)(3x + 8) \\ = 9x^2 + 48x + 64$$

## Factorising.....

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$x^2 + 2x - 8 = (x - 2)(x + 4)$$

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$2x^2 - 5x - 3 = (x - 3)(2x + 1)$$

$$15x^2 - 7x - 2 = (3x - 2)(5x + 1)$$

Match the graphs to their equations:

$$y = 3x + 1$$

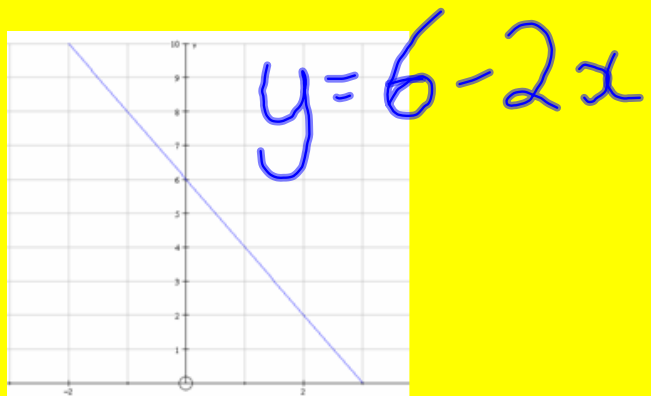
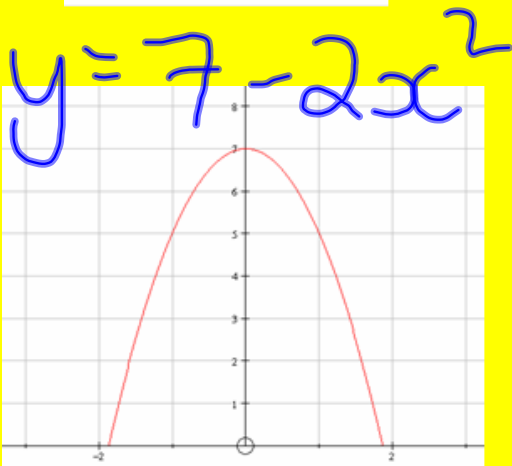
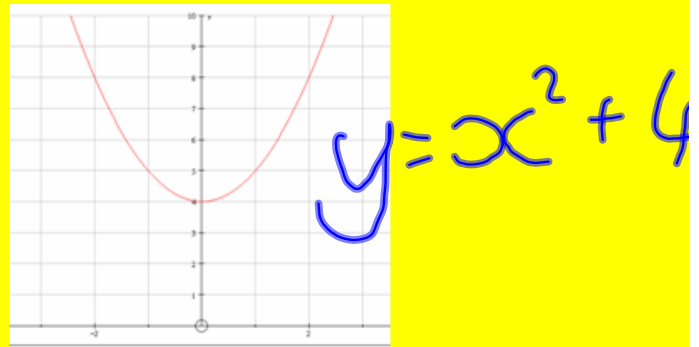
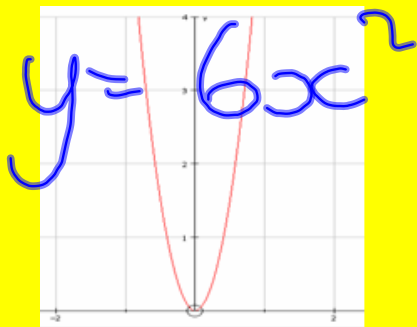
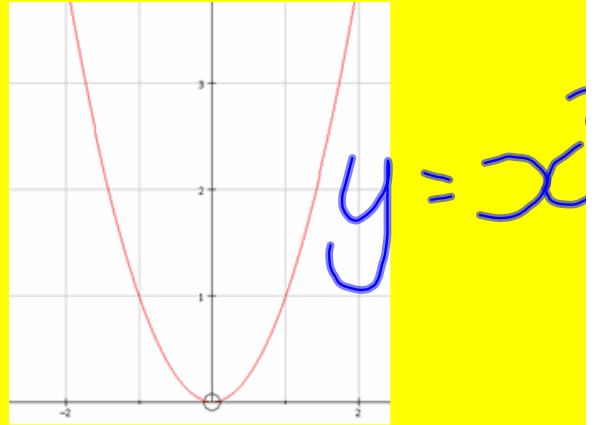
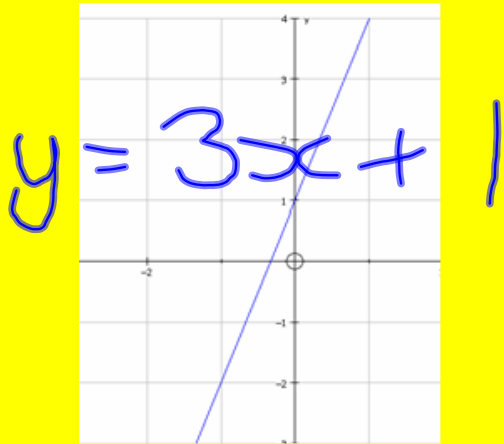
$$y = 6 - 2x$$

$$y = x^2$$

$$y = x^2 + 4$$

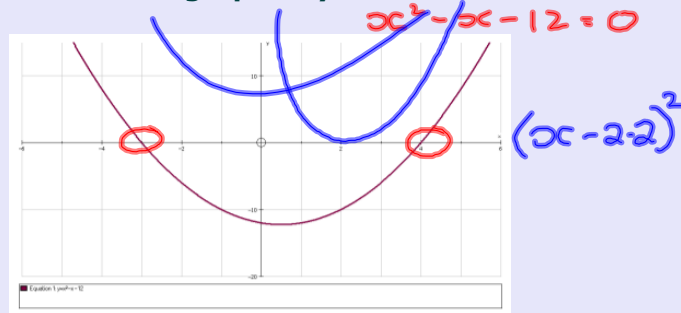
$$y = 7 - 2x^2$$

$$y = 6x^2$$



## Quadratic Graphs

Sketch the graph of  $y = x^2 - x - 12$



Solve the equation  $x^2 - x - 12 = 0$   $x = -3$   
or  $x = 4$

Factorise  $y = x^2 - x - 12$ .

$$y = (x-4)(x+3)$$

$$\begin{cases} x^2 - x - 12 = 0 \\ (x+4)(x+3) = 0 \end{cases}$$

$$x-4=0 \text{ or } x+3=0$$

$$x=4 \quad x=-3$$

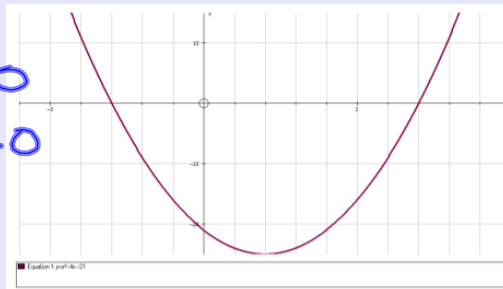
solve:

$$x^2 - 4x - 21 = 0$$

$$(x+3)(x-7) = 0$$

$$x+3=0 \text{ or } x-7=0$$

$$\underline{x=-3} \text{ or } \underline{x=7}$$



Solve:

$$(x+2)(x+8) = 0$$

$$x+2=0 \text{ or } x+8=0$$

$$x=-2 \text{ or } x=-8$$

$$(x-5)(x+6) = 0$$

$$x-5=0 \text{ or } x+6=0$$

$$x=5 \text{ or } \underline{x=-6}$$

$$(x+2)(2x-9) = 0$$

$$x+2=0 \text{ or } 2x-9=0$$

$$x=-2 \text{ or } 2x=9 \quad x=4.5$$

$$x^2 + 8x + 15 = 0$$

$$x^2 - 12x - 14 = 0$$

## Solving Quadratic Equations by factorising

**Factorise, and hence solve:**

$$x^2 - 4x - 21 = 0$$

$$x^2 - 11x + 30 = 0$$

$$2x^2 + 9x - 5 = 0$$

**p 336**

**A1 middle column only**

**A2 middle column only**

**A3 and A5**

**page 341**

**D1, D2, D3 middle columns only.**

**D4, 5, 6**



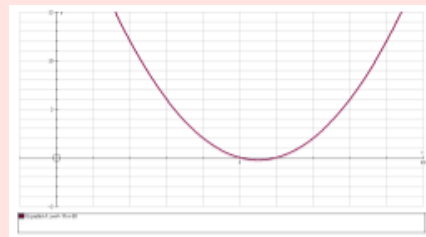
## Solving Quadratic Equations by factorising

Factorise, and hence solve:

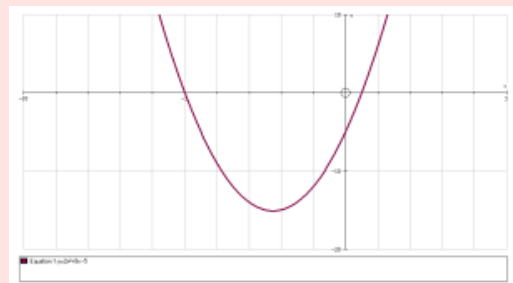
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p 336

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page 341

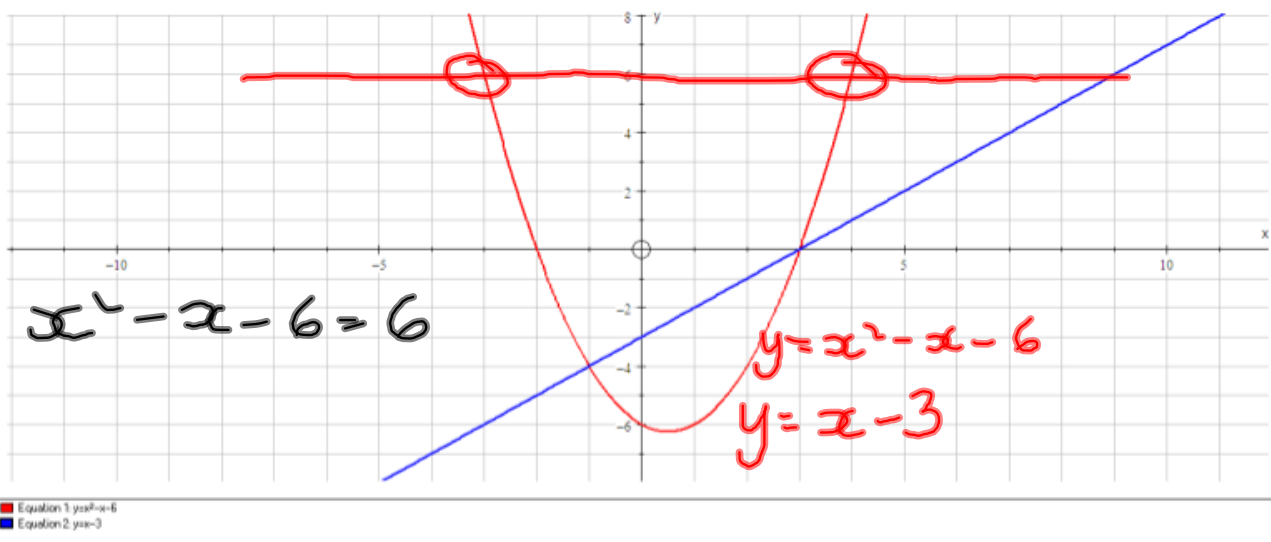
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D4,5,6

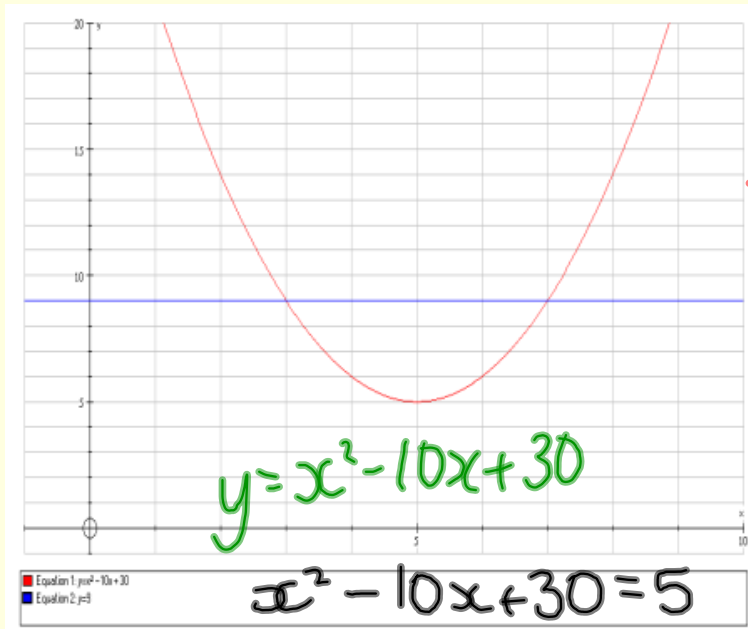
## Simultaneous equations (linear/quadratic)

On one pair of axes plot the graphs  
 $y=x^2-x-6$  and  $y=x-3$

$$\begin{aligned}x-3 &= x^2-x-6 \\ 0 &= x^2-2x-3 \\ 0 &= (x-3)(x+1) \\ x &= 3 \text{ or } x = -1\end{aligned}$$



## Quadratic Simultaneous Equations solved by drawing graphs.



$$x^2 - 10x + 30 = 5$$
$$x = 5 \text{ twice}$$

$x^2 - 10x + 30 = 0$   
There are no  
real solutions.

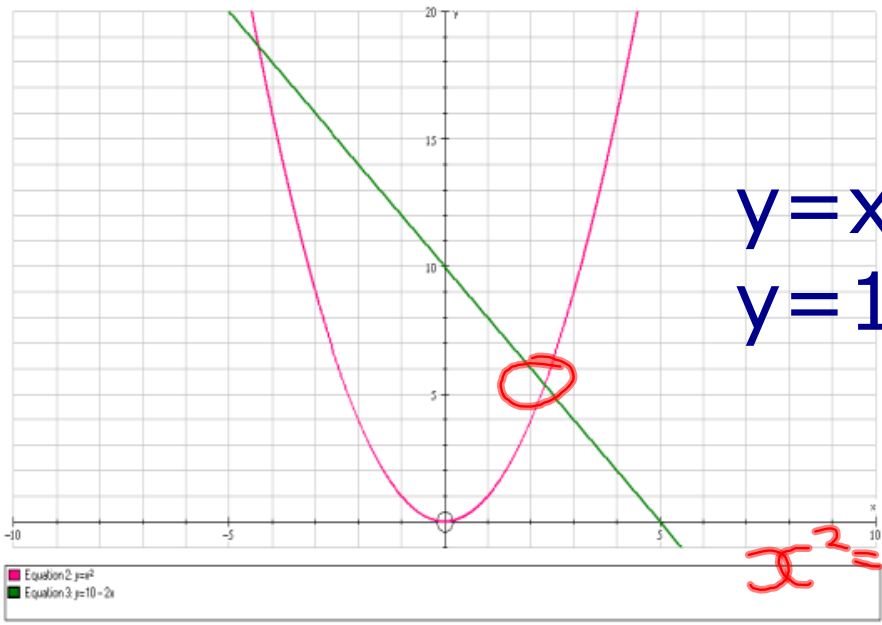
$x^2 - 10x + 30 = 9$   
 $x = 3 \text{ or } 7$

$x^2 - 10x + 21 = 0$   
 $x = 3 \text{ or } 7$



$$y = x^2 - 10x + 30$$
$$y = x^2$$

$$x = 3$$
$$y = 9$$



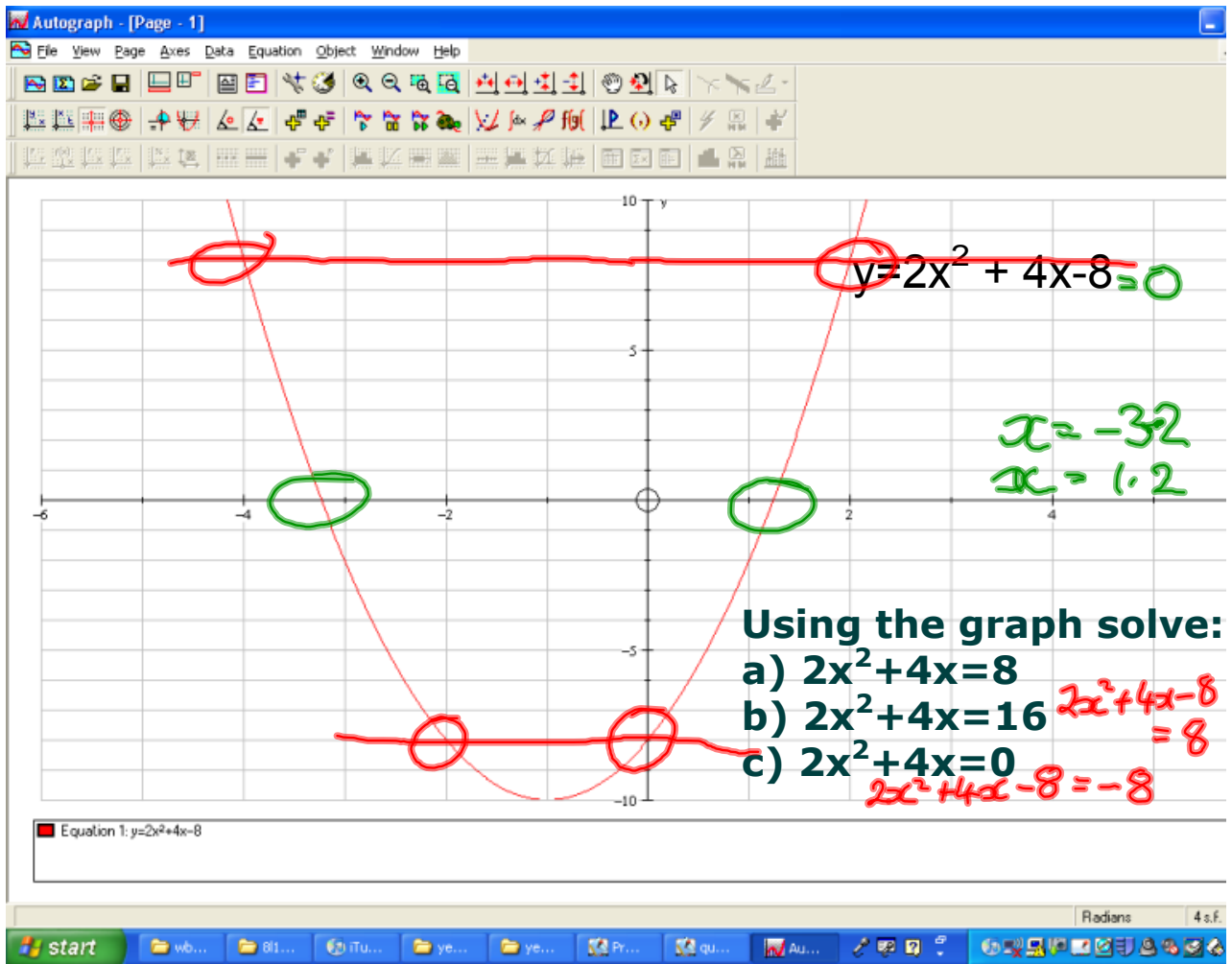
$$x = -4.2$$
$$y = 18.6$$

$$y = x^2$$

$$y = 10 - 2x$$

$$x = 2.3$$
$$y = 5.4$$

$$x^2 = 10 - 2x$$



$$11 \text{ a) } x^2 = 5x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x-3=0 \text{ or } x-2=0$$

$$x=3 \text{ or } 2$$

$$c) \quad y = 5x^2 + 4x - 3 \text{ and } y = 5 - 2x$$

$$2x + y = 5$$

$$5 - 2x = 5x^2 + 4x - 3$$

$$5x^2 + 6x - 8 = 0$$

$$(5x-4)(x+2) = 0$$

$$x = \frac{4}{5} \text{ or } x = -2$$

$$y = 5 - 2x : \quad y = 5 - \frac{8}{5} \quad y = 5 + 4$$

$$= 3\frac{2}{5}$$

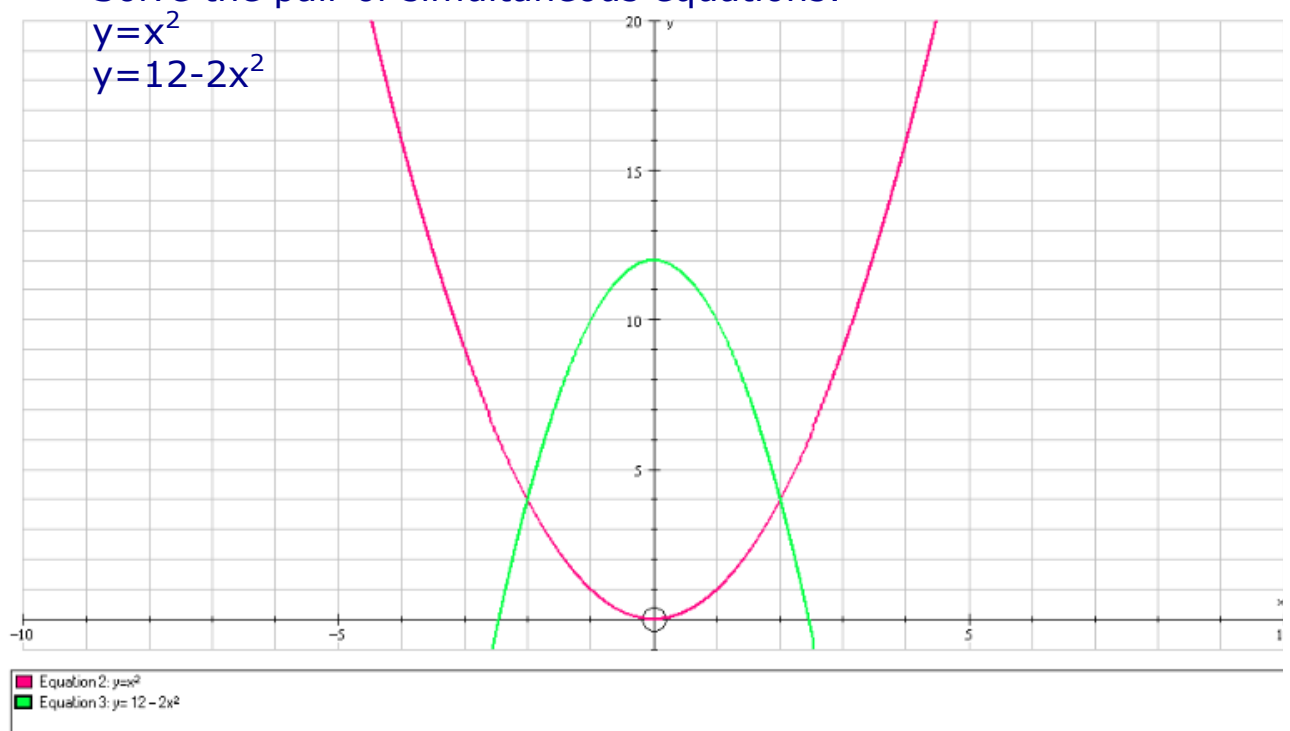
$$y = 9$$

11 b

Solve the pair of simultaneous equations:

$$y = x^2$$

$$y = 12 - 2x^2$$



page 347

**Homework:**

**On graph paper draw the graph of  $y=4x^2-2x-20$ .**

**Using your graph solve:**

**a)  $4x^2-2x-20=0$**

**b)  $4x^2-2x-16=0$**

**c)  $4x^2-2x = 6$**

**Prove, using algebra, that your solutions to part a) are correct.**

**Extension: Are the solutions to  $4x^2-2x-20=0$  and  $2x^2-x-10=0$  the same? Explain your answer graphically, algebraically or even better, both!**

### Completing the Square

This approach can be used to solve some quadratic equations that do not easily factorise.

Solve:

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$x-4=0 \text{ or } x=4 \text{ (twice)}$$

Perfect squares:

$$(x-8)^2 = x^2 - 16x + 64$$

p 342 e2

$$(x-3)^2 = x^2 - 6x + 9$$

Is  $x^2 - 6x + 7$  a perfect square?

To solve  $x^2 - 6x + 7 = 0$  using perfect squares:

$$x^2 - 6x = -7$$

$$\frac{x^2 - 6x + 9 - 9 = -7}{(x-3)^2 - 9 = -7}$$

$$(x-3)^2 = 2$$

$$x-3 = \pm\sqrt{2}$$

$$x = \sqrt{2} + 3 \text{ or } x = -\sqrt{2} + 3$$

$$x^2 - 6x + 9 = 2$$

$$(x-3)^2 = 2$$

$$x-3 = \pm\sqrt{2}$$

$$x = \pm\sqrt{2} + 3$$

Try:

Solve  $x^2 + 8x - 10 = 0$  by completing the square

$$(x+4)^2 = x^2 + 8x + 16$$

$$\boxed{x^2 + 8x} - 10 = 0$$

$$\boxed{(x+4)^2 - 16} - 10 = 0$$

$$(x+4)^2 - 26 = 0$$

$$(x+4)^2 = 26$$

$$x+4 = \pm\sqrt{26}$$

Solve  $x^2 - 10x + 30 = 0$  by completing the square.

$$x = \sqrt{26} - 4 \text{ or } x = -\sqrt{26} - 4$$

$$\boxed{(x-5)^2 = x^2 - 10x + 25}$$

$$(x-5)^2 = -5$$

There are no real solutions

Solve by completing the square:

$$x^2 + 14x - 5 = 0$$

$$\boxed{(x+7)^2 = x^2 + 14x + 49}$$

$$(x+7)^2 = 54$$

$$x+7 = \pm\sqrt{54}$$

$$x = \pm\sqrt{54} - 7$$

$$x = \pm 3\sqrt{6} - 7$$

(Harder) Completing the square

$$x^2 - 4x = 3$$

$$(x-2)^2 = x^2 - 4x + 4$$

$$(x-2)^2 - 4 = 3$$

$$(x-2)^2 = 7$$

$$x-2 = \pm\sqrt{7}$$

$$x = \pm\sqrt{7} + 2$$

$$2y^2 + 4y = 7$$

$$y^2 + 2y = \frac{7}{2}$$

$$(y+1)^2 - 1 = \frac{7}{2}$$

$$(y+1)^2 = 4.5$$

$$y+1 = \pm\sqrt{4.5}$$

$$(y+1)^2 = y^2 + 2y + 1$$

$$y = \pm\sqrt{4.5} - 1$$

$$y = \pm\sqrt{\frac{9}{2}} - 1$$

$$2x^2 - 3x - 3 = 0$$

$$x^2 - \frac{3x}{2} = \frac{3}{2}$$

$$\left(x - \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3}{2}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{3}{2} + \frac{9}{16} = \frac{24}{16} + \frac{9}{16} = \frac{33}{16}$$

$$\left(x - \frac{3}{4}\right) = \pm\sqrt{\frac{33}{16}}$$

$$\left(x - \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$$

$$= x^2 - \frac{3}{2}x + \frac{9}{16}$$

$$x = \frac{\pm\sqrt{33}}{4} + \frac{3}{4}$$

$$4y^2 - y = 8$$

$$y^2 - \frac{y}{4} = 2$$

$$\left(y - \frac{1}{8}\right)^2 - \frac{1}{64} = 2$$

$$\left(y - \frac{1}{8}\right)^2 = 2 + \frac{1}{64}$$

$$y - \frac{1}{8} = \pm\sqrt{\frac{129}{64}} = \frac{\pm\sqrt{129}}{8}$$

$$y = \frac{\pm\sqrt{129}}{8} + \frac{1}{8} = \frac{1 \pm \sqrt{129}}{8}$$

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$y^2 - 6y + 1$$

$$(y-3)^2$$

## Solving Quadratic Equations using The Formula.

All quadratics can be written in the form:

$$y = ax^2 + bx + c$$

$$4x^2 + 7x - 9 = 0$$

$$5x^2 - 6 = 0$$

$$a = 5$$
$$b = 0$$
$$c = -6$$

$$x^2 - 4x = 11$$

$$4x^2 - 7x = 0$$

$$a = 4$$
$$b = -7$$
$$c = 0$$

$$a = 1$$
$$b = -4$$
$$c = -11$$

Solve the equation:

$$2x^2 + 5x - 3 = 0$$

$$(2x-1)(x+3) = 0$$

$$2x-1=0 \text{ or } x+3=0$$

$$2x=1$$

$$x = -3$$

$$\text{or}$$
$$x = +\frac{1}{2}$$

The Formula gives solutions to any quadratic that has solutions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

$$2x^2 + 5x - 3 = 0$$

$$a = 2 \quad b = 5 \quad c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - (-24)}}{4}$$

$$x = \frac{-5 \pm \sqrt{49}}{4}$$

page 344

$$x = \frac{-5+7}{4} \text{ or } x = \frac{-5-7}{4}$$

$$x = \frac{2}{4} = \frac{1}{2}$$

$$x = \frac{-12}{4} = -3$$

## Practising the Formula

Solve the equation:

$$3x^2 + 2x = 7 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$3x^2 + 2x - 7 = 0$$

$$a = 3, \quad b = 2 \quad c = -7$$

$$x = \frac{-2 \pm \sqrt{4 - (-84)}}{6}$$

$$= \frac{-2 \pm \sqrt{88}}{6}$$

$$x = \frac{-2 + \sqrt{88}}{6} \quad \text{or} \quad x = \frac{-2 - \sqrt{88}}{6}$$

$$= 1.23 \quad \text{or} \quad -1.90 \quad \text{to 2dp}$$

$$x^2 - 3x - 1 = 0 \quad a = 1, b = -3, c = -1$$

$$x = \frac{+3 \pm \sqrt{9 - (-4)}}{2a}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$$x = 3.30$$
$$\text{or } x = -0.30$$

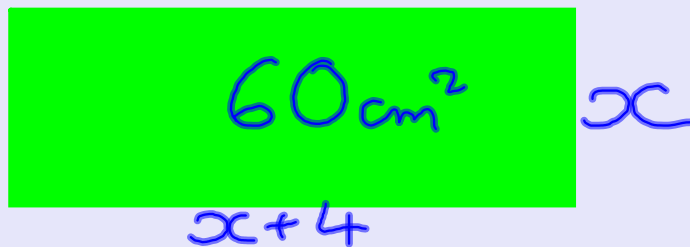
$$\sqrt{b^2 - 4ac} = \pm \sqrt{64}$$

↑  
perfect sq.

$$(x + \quad)^2 = \sqrt{\textcircled{16}} \leftarrow \text{perfect square}$$

Factorising

**Forming Quadratic Equations and solving them from practical situations.**



**A rectangle has area 60cm<sup>2</sup>. Its length is 4cm more than its width. Find the width.**

$$\begin{aligned}x(x+4) &= 60 \\x^2 + 4x &= 60 \\x^2 + 4x - 60 &= 0 \\(x+10)(x-6) &= 0 \\x &= -10 \text{ or } x = 6\end{aligned}$$

width  
is 6cm  
|

**In the next questions make an equation and solve it.**

**Page 202 C7 - C11**

C7  
and C10

**page 205-206 E3 onwards**

Using the general equation  
 $y = ax^2 + bx + c$  and by  
 completing the square prove  
 The Formula

$$ax^2 + bx + c = 0 \quad \frac{3 \times 4}{2}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left[ \left(x + \frac{b}{2a}\right)^2 = x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} \right]$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \cdot \frac{c}{a} = \frac{4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} + \frac{\pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$