

## Equivalent fractions

How many fractions are there equivalent to :

1)  $\frac{1}{5} = \frac{2}{10} = \frac{4}{20} = \frac{6}{30} = \frac{8}{40} = \frac{100}{500}$

2)  $\frac{2}{5} = \frac{4}{10} = \frac{8}{20} = \frac{10}{25} = \frac{6}{15}$

Page 90 A2-A4

## Cancelling Fractions

$$\frac{\cancel{4}^1}{\cancel{12}_3} = \frac{1}{3}$$

Cancel these fractions to their lowest terms

$$\frac{20}{45} = \frac{4}{9} \quad \frac{5}{45} = \frac{1}{9} \quad \frac{40}{90} = \frac{4}{9}$$

$$\frac{\cancel{27}^9}{\cancel{42}_{14}} = \frac{9}{14} \quad \frac{16}{96} = \frac{1}{6} \quad \frac{120}{600} = \frac{1}{5}$$

$\frac{2}{6}$     $\frac{1}{3}$     $\frac{12}{36}$   
 $\frac{102}{404}$

$\frac{10}{16}$     $\frac{60}{96}$     $\frac{30}{48} = \frac{5}{8}$   
 $\frac{15}{32}$     $\frac{5}{8}$

Which of these fractions is the odd man out?

$\frac{22}{77} = \frac{2}{7}$     $\frac{12}{42} = \frac{6}{21} = \frac{2}{7}$   
 $\frac{7}{17}$     $\frac{2}{7}$

Ordering fractions

There are two main ways to order fractions:

- if you can use a calculator, change them to decimals.
- use equivalent fractions to compare.

Put these fractions in order of size, starting with the smallest:

1)  $\frac{5}{6}$     $\frac{2}{3}$     $\frac{3}{10}$     $\frac{1}{2}$     $\frac{3}{10}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$   
 no calculator

2)  $\frac{4}{33}$     $\frac{6}{11}$     $\frac{1}{2}$     $\frac{5}{6}$     $\frac{4}{33}, \frac{1}{2}, \frac{6}{11}, \frac{5}{6}$   
 no calculator

3)  $\frac{1}{4}$     $\frac{2}{5}$     $\frac{1}{6}$     $\frac{2}{7}$     $\frac{1}{6}, \frac{2}{7}, \frac{1}{4}, \frac{2}{5}$

4)  $\frac{5}{8}$     $\frac{3}{7}$     $\frac{4}{9}$     $\frac{3}{10}$     $\frac{3}{10}, \frac{3}{7}, \frac{4}{9}, \frac{5}{8}$

5)  $\frac{7}{11}$     $\frac{4}{15}$     $\frac{6}{12}$     $\frac{5}{7}$

6)  $\frac{1}{6}$     $\frac{2}{9}$     $\frac{5}{12}$     $\frac{7}{15}$

Dolphin game

Kyle eats  $\frac{2}{7}$  of a 12" pizza.

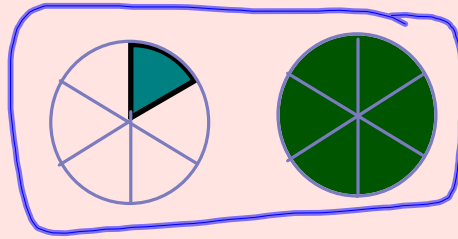
Dom eats  $\frac{3}{10}$  of a 12" pizza.

Who eats most?

$\frac{20}{70}$

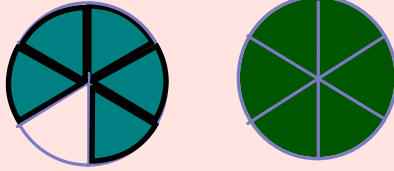
$\frac{21}{70}$

Improper Fractions ↔ Mixed numbers

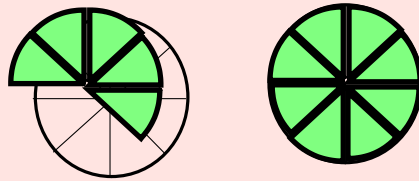


$$1\frac{1}{6} = \frac{7}{6}$$

mixed no.      Improper/topheavy fractions



$$1\frac{5}{6} = 1\frac{5}{6}$$




$$1\frac{5}{8} = \frac{13}{8}$$

Draw a diagram to show  $1\frac{2}{3} = \frac{5}{3}$

Draw a diagram to show  $\frac{7}{5} = 1\frac{2}{5}$

$$\frac{5}{5} = 1$$

American currency is in cents:

dimes (10c), 

quarters (25c) 

and dollars (100c).



If I have 11 quarters, how much money is that?

Finding a fraction of an amount.

$$\frac{5}{8} \text{ of } 88 = 55$$

means find one eighth, then find 5 of those

work out:

$$\frac{3}{7} \text{ of } 35 = 15$$

$$\frac{4}{5} \text{ of } 45 = 36$$

$$\frac{2}{9} \text{ of } 36 = 8$$

A bag of 80 smarties is shared among 4 people so that

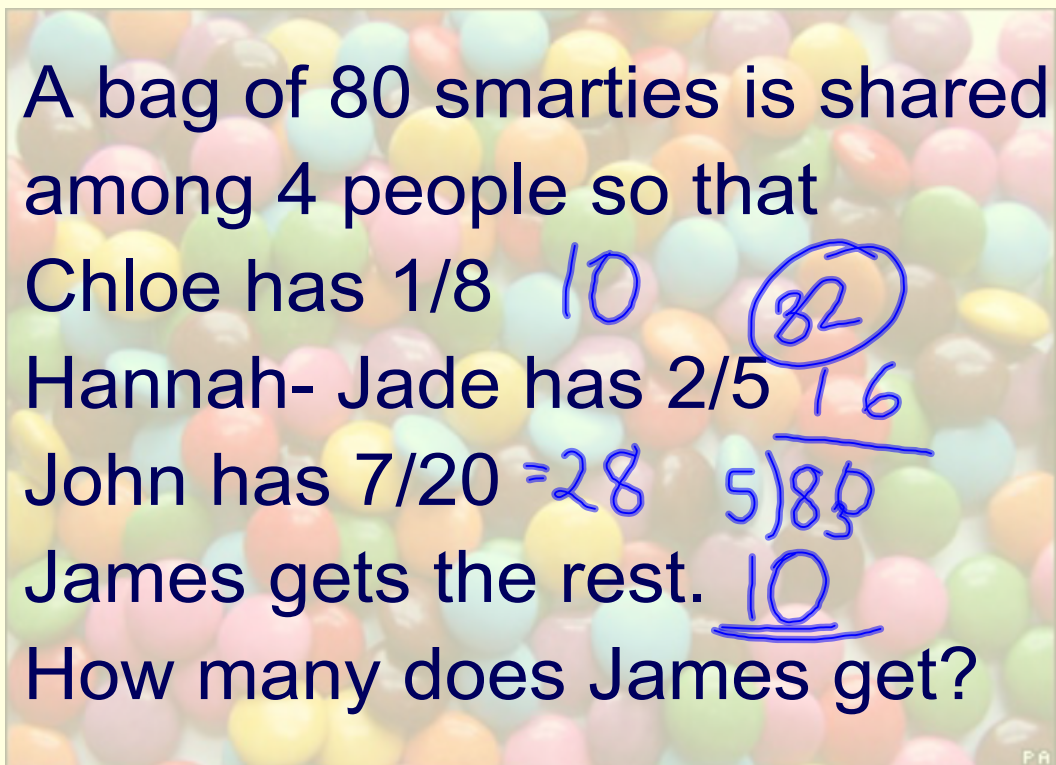
Chloe has  $\frac{1}{8}$  10

Hannah- Jade has  $\frac{2}{5}$  16

John has  $\frac{7}{20} = 28$

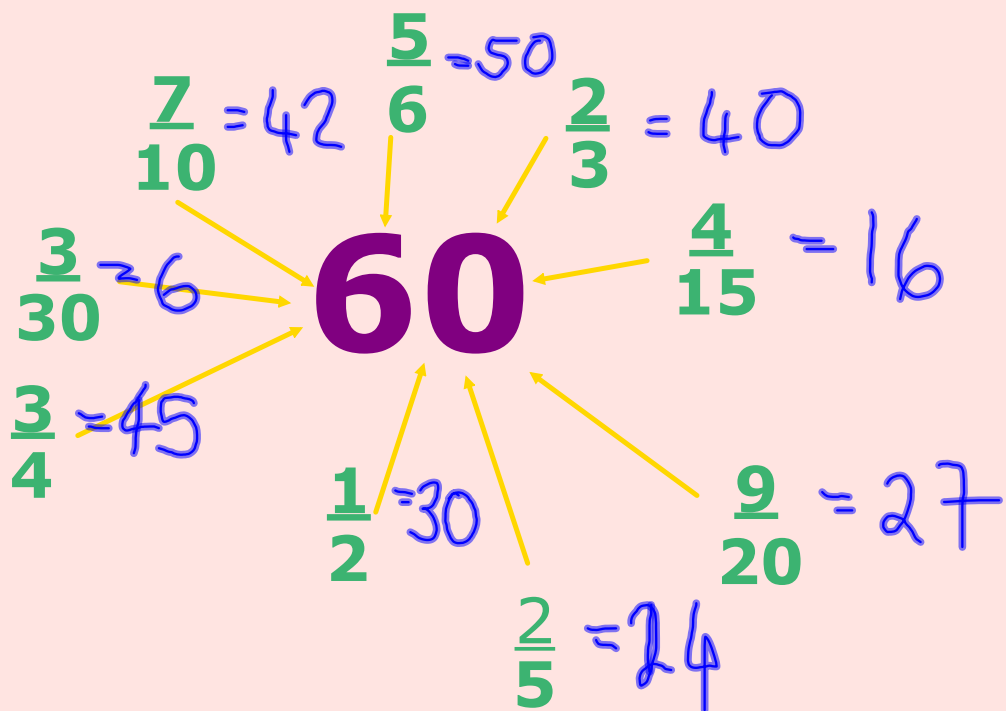
James gets the rest. 10

How many does James get?



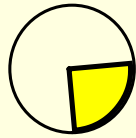
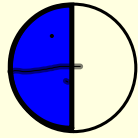
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Work out:

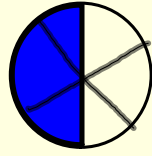
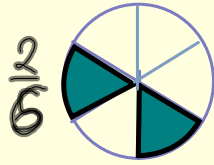


page 145 A2, B2-B8 evens only, C1-C4

Adding Fractions with different denominators



$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$



$$\frac{1}{2} + \frac{1}{2} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

$$\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$

$$\frac{1}{3} + \frac{1}{6}$$

$$1. \quad \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$2. \quad \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

$$3. \quad \frac{3}{10} + \frac{2}{5} = \frac{7}{10}$$

$$\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$4. \quad \frac{2}{5} + \frac{3}{15} = \frac{9}{15}$$

$$5. \quad \frac{3}{4} + \frac{1}{12} = \frac{10}{12} = \frac{5}{6}$$

$$6. \quad \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$\frac{2}{5} + \frac{3}{7} =$$

How to add and subtract fractions without going to pieces

1. Find a common denominator, the lower the better.
2. Using your common denominator make equivalent fractions.
3. Add (or subtract) the numerators.
4. Simplify.

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$
$$\frac{2}{5} + \frac{1}{8} = \frac{16+5}{40} = \frac{21}{40}$$

$$\frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$$
$$\frac{3}{5} + \frac{5}{9} = \frac{27+25}{45}$$

$$\frac{2}{3} + \frac{1}{5} = \frac{10+3}{15} = \frac{13}{15}$$
$$\frac{5}{6} + \frac{1}{9} = \frac{52}{45} = 1\frac{7}{45}$$

$$= \frac{15+2}{18} = \frac{17}{18}$$

Isabelle eats  $\frac{3}{8}$  of a bar of  
8

chocolate. Lorna eats  $\frac{2}{5}$  of the  
bar? 5

How much is left for Oliver?

$$\frac{3}{8} + \frac{2}{5} = \frac{15+16}{40} = \frac{31}{40}$$

Oliver had  $\frac{9}{40}$

$$\frac{2}{5} + \frac{1}{2} + \frac{1}{10}$$

$$\frac{1}{4} + \frac{1}{3} + \frac{5}{12}$$

Find 3 fractions that have a sum of 1,  
but all have different denominators.

$$\frac{1}{3} + \frac{2}{6} + \frac{3}{12}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{2}{8}$$

$$\frac{13}{26} + \frac{14}{52} + \frac{3}{13}$$

$$\frac{2}{4} + \frac{1}{3} + \frac{2}{12}$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

## Cancel when you can

$$\frac{12}{20} = \frac{3}{5}$$

$$\frac{12}{20} = \frac{\cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 5} = \frac{3}{5}$$

Write the following as product of their primes and cancel:

$$\frac{8}{12} = \frac{\cancel{2} \times \cancel{2} \times 2}{\cancel{2} \times \cancel{2} \times 3} = \frac{2}{3}$$

$$\frac{24}{42} = \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 7} = \frac{4}{7}$$

### Multiplying Fractions

This is the easiest of all the fraction questions.  
Multiply the numerators together.  
Multiply the denominators together.  
That's it!

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21} \quad \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

If you have a whole number, change it to an improper fraction:

$$2\frac{1}{3} \times \frac{3}{4} = \frac{7}{3} \times \frac{3}{4} = \frac{7}{4} = 1\frac{3}{4}$$

A shortcut:

$$\frac{2}{3} \times \frac{6}{10} = \frac{2 \times 2 \times 3}{3 \times 2 \times 5} = \frac{2}{5}$$

$$\frac{3}{20} \times \frac{5}{9} = \frac{3 \times 5}{2 \times 2 \times 5 \times 3 \times 3} = \frac{1}{12}$$

$$\frac{13}{20} \times \frac{5}{39} = \frac{1}{12} \quad \text{by HI}$$

Cancel where you can

$$\frac{45}{70} = \frac{\cancel{5} \times 9}{7 \times \cancel{2} \times 5} = \frac{9}{14}$$

$$\frac{45}{120} = \frac{\cancel{5} \times \cancel{3} \times 3}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5}$$

$$\frac{42}{70} = \frac{\cancel{7} \times 2 \times 3}{\cancel{7} \times 2 \times 5} = \frac{3}{5}$$

$$\frac{39}{390} = \frac{\cancel{39}}{\cancel{39} \times 10} = \frac{1}{10}$$

$$\frac{450}{720} = \frac{\cancel{10} \times \cancel{5} \times 9}{\cancel{10} \times \cancel{8} \times 9}$$

page 150 E1-E6, F1 - F8

$$\frac{\cancel{2}^1 \times \frac{5}{\cancel{6}_3} \times \frac{\cancel{3}^1}{\cancel{7}_1} \times \frac{\cancel{14}^2}{27}}{81} = \frac{10}{81}$$

$$\frac{2 \times 5 \times 3 \times 14}{3 \times 6 \times 7 \times 27}$$

$$\frac{\cancel{2}^1 \times \cancel{5}^1 \times \cancel{6}^2 \times \cancel{42}^{217}}{\cancel{36}^3 \times \cancel{8}^4 \times \cancel{27}^9 \times \cancel{10}^2} = \frac{7}{36}$$

$$\frac{2520}{12960}$$

$$\frac{42}{8} = \frac{21}{4}$$

$$\frac{\cancel{2}^2 \times \cancel{6}^1 \times \cancel{8}^2 \times \cancel{20}^1 \times \cancel{7}^1}{\cancel{16}^4 \times \cancel{14}^2 \times \cancel{15}^3 \times \cancel{3}^1} = \frac{2}{3}$$

$$\begin{array}{r}
 \frac{\cancel{1}^3 \cancel{3} \times \cancel{12}^3 \times \cancel{10}^5 \times \cancel{35}^7}{\cancel{25}^5 \times \cancel{16}^4 \times \cancel{9}^3 \times \cancel{28}^4} = \frac{1}{8} \\
 \frac{\cancel{12}^4 \times \cancel{18}^3 \times \cancel{9}^1 \times \cancel{100}^4}{\cancel{75}^3 \times \cancel{27}^3 \times 45 \times 6} = \frac{16}{45}
 \end{array}$$

## Dividing by a fraction

This is easy if you can remember the rule:  
Turn the second fraction upside down and multiply!

$$\frac{1}{4} \div \frac{1}{4} = \frac{1}{4} \times \frac{4}{1} = \frac{4}{4} = 1$$

How many quarters are there in a half?

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

$$\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$$

page 284 C1, C2

$$\begin{array}{ccc} \frac{1}{2} \div \frac{1}{8} & \frac{1}{5} \div \frac{1}{10} & \frac{3}{4} \div \frac{1}{4} \\ = \frac{1}{2} \times \frac{8}{1} = 4 & \frac{1}{5} \times \frac{10}{1} = 2 & \frac{3}{4} \times \frac{4}{1} = 3 \end{array}$$

$$\begin{array}{ccc} \frac{5}{6} \div \frac{1}{12} & \frac{3}{7} \div \frac{6}{28} \\ \frac{5}{6} \times \frac{12}{1} = 10 & \frac{3}{7} \times \frac{28}{6} = 2 \end{array}$$

page 284 C3- C9, D1- D2

$$5 \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$$

$$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3} = \frac{8}{3} = 2\frac{2}{3}$$

C 2 → evens only  
D1 + D2

## Converting fractions to decimals

Which fractions do you know the decimal equivalent of?

Terminating decimals	Recurring Decimals
$\frac{1}{2} = 0.5$ $\frac{1}{5} = 0.2$ $\frac{3}{4} = 0.75$ $\frac{1}{10} = 0.1$ <span style="color: red;">2, 4, 8, 16</span> <span style="color: red;">10,</span>	$\frac{2}{3} = 0.\dot{6}$ $\frac{1}{9} = 0.\dot{1}$ $\frac{1}{3} = 0.\dot{3}$ $\frac{1}{11} = 0.\dot{0}9$ $\frac{1}{9} = 0.\dot{1}$ $\frac{5}{12} = 0.41\dot{6}$ $\frac{1}{6} = 0.1\dot{6}$ $\frac{10}{15} = 0.76\dot{9}23$ <span style="color: red;">odd number</span> <span style="color: red;">denoms recur</span> <span style="color: red;">except 5</span> <span style="color: blue;">(15)</span> $\frac{5}{7} = 0.71428\dot{5}$



Investigate fractions with a denominator of 7

Investigate fractions with a denominator of 9.

Try to find a rule for fractions which terminate.  
Test it out on the fractions:

Converting fractions to decimals by hand

$$\frac{1}{12} = 1 \div 12 \quad 0.0833 \quad \frac{1}{12} = 0.\dot{0}83$$

$$\frac{5}{6} = 5 \div 6 \quad 0.83 \quad \frac{5}{6} = 0.8\dot{3}$$

$$\frac{4}{9} = 0.4 \quad 9 \overline{) 4.0000}$$

$12 \overline{) 1.00000000}$   
 $\quad 10 \quad 4 \quad 4 \quad 4$

Can this recurring decimal be expressed as a fraction?

$$0.34343434\dots$$

$$\begin{aligned} \text{Let } x &= 0.343434\dots \\ 100x &= 34.343434\dots \\ x &= 0.343434\dots \end{aligned} \left. \vphantom{\begin{aligned} \text{Let } x &= 0.343434\dots \\ 100x &= 34.343434\dots \\ x &= 0.343434\dots \end{aligned}} \right\} -$$

$$99x = 34$$

$$x = \frac{34}{99}$$

$$0.4545454545\dots$$

$$\begin{aligned} \text{Let } x &= 0.454545\dots \\ 100x &= 45.454545\dots \\ x &= 0.454545\dots \end{aligned} \left. \vphantom{\begin{aligned} \text{Let } x &= 0.454545\dots \\ 100x &= 45.454545\dots \\ x &= 0.454545\dots \end{aligned}} \right\} -$$

$$99x = 45$$

$$x = \frac{45}{99} = \frac{15}{33} = \frac{5}{11}$$

$$0.678678678\dots$$

$$\text{Let } x = 3.153153153\dots$$

$$\begin{aligned} 1000x &= 3153.153 \\ x &= 3.153 \end{aligned} \left. \vphantom{\begin{aligned} 1000x &= 3153.153 \\ x &= 3.153 \end{aligned}} \right\} -$$

$$7.405140514051\dots$$

$$999x = 3150$$

$$x = \frac{3150}{999} = \frac{1050}{333} = \frac{350}{111} = 3\frac{17}{111}$$

Convert to a fraction:

a)  $0.414141\dots$   $x = \frac{41}{99}$

b)  $0.787878\dots$   $\frac{78}{99} = \frac{26}{33}$

c)  $0.213213213$   $\frac{213}{999} = \frac{71}{333}$

Recurring Decimals to Fractions

Express 3.5555... as a fraction

$$\begin{array}{r} \text{Let } x = 3.5555 \\ 10x = 35.5555 \\ \hline x = 3.5555 \\ \hline 9x = 32 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} -$$
$$x = \frac{32}{9} = 3\frac{5}{9}$$

Examples:

Express as a fraction:

1)  $0.373737\dots = \frac{37}{99}$

2) 5.676767

3) 8.76876876876

4) 23.4444444

$$\begin{array}{r} \text{Let } x = 0.373737 \\ 100x = 37.373737 \\ \hline x = 0.373737 \\ \hline 99x = 37 \\ \hline x = \frac{37}{99} \end{array}$$

Show that  $5.676767 = 5\frac{67}{99}$

$$\begin{array}{r} \text{Let } x = 5.6767\dots \\ 100x = 567.6767 \\ \hline x = 5.6767 \\ \hline 99x = 562 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} -$$
$$x = \frac{562}{99} = 5\frac{67}{99}$$

Show that  $0.546546546 = \frac{182}{333}$

$$\begin{array}{r} \text{Let } x = 0.546546\dots \\ 1000x = 546.546546\dots \\ \hline x = 0.546546 \\ \hline 999x = 546 \\ \hline x = \frac{546}{999} = \frac{182}{333} \end{array}$$

Express a)  $0.\overline{34}$  as a fraction

b)  $3.\overline{728}$  as a fraction

$$\text{let } x = 5.676767$$

$$\begin{array}{r} 100x = 567.676767 \\ x = 5.676767 \\ \hline 99x = 562 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} -$$

$5 \times 99 = 495$

$$x = \frac{562}{99} = 5 \frac{67}{99} x.$$

$$3) 8.76876876$$

$$\begin{array}{r} \text{let } x = 8.76876876 \\ 1000x = 8768.76876876 \\ x = 8.76876876 \\ \hline 999x = 8760 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} -$$

$$x = \frac{8760}{999} = 8 \frac{768}{999} = 8 \frac{256}{333}$$

$8 \times 999 = 8000 - 8$

$$\begin{array}{r} \text{let } x = 23.\dot{4} \\ 10x = 234.\dot{4} \\ x = 23.\dot{4} \\ \hline \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} -$$

$$9x = 211$$

$$x = \frac{211}{9} = 23 \frac{4}{9}$$

$$1) 0.838383 \dots \quad x = \frac{83}{99}$$

$$2) 1.\dot{2} \quad x = 1 \frac{2}{9}$$

$$3) 0.12341234 \quad x = \frac{1234}{9999}$$

Decide whether these statements are true or false.  
You need to be able to defend your opinion.

20% is more  
than 10%

If I borrow £100  
on a credit card  
and pay 5% per  
month interest,  
after 3 months I  
will owe £115.

A 20% reduction  
in price is the same  
as 80% of the  
original price.

$$10\% + 10\% = 20\%$$

60% is the  
same as  
three fifths.

$\frac{5}{6}$  of a school day is spent in lessons.

What percentage of the day is spent in lessons?

72% of students like sport.

What fraction of the students like sport?

Converting between fractions and percentages

Fraction to percent:

Percent to fraction:

### Solving problems with fractions.

1. A cake is divided up between 3 people. Jess has two fifths, Nic has two sevenths and Jamie has the rest.

- How much does Jamie get?
- Who has the biggest piece?
- If Jamie gave half his piece to Nic how much would they each get?

2. A bag of sweets is shared between 4 people. Toby has 20%, Cameron has one fifteenth, Trinity has one third and Alice has 60 sweets. How many sweets did they have?

3. A school day of 6 hours is divided into modules of three-quarters of an hour. How many modules are there in a day?

8<sup>?</sup>

## B.O.G.O.F

**I paid £ 60 for my shopping bill.**

**Later, when I looked at my receipt, it showed a “multibuy” saving of £20.**

**This means about 33% of my food was free.**

**On a different occasion I spent £64.55 and my multibuy saving was £18. What percentage of my food was free?**

# THE SALES

I got a great bargain in the sales. This bag was £15.00 last week. Today it was reduced to £5.00.

Karen

Well, what do you think of this... I spent £30.00 on these trousers and they were priced at £50.00. That's an even better bargain.

Sharon

Page 109, 110 C1 - D7

understand equivalent fractions, simplifying a fraction by cancelling all common factors  
order fractions by rewriting them with a common denominator  
calculate a given fraction of a given quantity, expressing the answer as a fraction  
express a given number as a fraction of another  
add and subtract fractions by writing them with a common denominator  
perform short division to convert a simple fraction to a decimal  
addition, subtraction, multiplication and division of mixed numbers  
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction  
distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals)  
convert a recurring decimal to a fraction  
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction  
convert simple fractions of a whole to percentages of the whole and vice versa  
use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer